Prof. C. Moser
Prof. T.J. Kippenberg
Fall Term 2024

Laser : Theory and Modern Applications

ECOLE POLYTECHNIQUE FEDERALE DE LAUSANNE (EPFL)

Homework No. 4

4.1 Optimal output power of a laser For a laser cavity there is a trade-off between the reflectivity of the mirror and the amount of output power possible, high reflectivities will lead to low threshold but low output power, while low reflectivity will increase the pump threshold. Assume that the mirror has a scattering loss (s) and a transmission (t), that needs to be optimized. Derive the optimal transmission (t_{opt}) and the maximum possible output intensity I_ν^{out}.

4.2 Laser output power with spontaneous emission An actual laser has even below threshold power that is emitted. This can be explained via the effect of spontaneous emission.

— To incorporate the spontaneous emission into our photon rate equations, re-write the rate equations not for the photon flux (Φ_ν) but for the number of in-tracavity photons ($\bar{n}_\mathrm{c}$). Once re-written, the spontaneous emission can be incorporated by modifying the decay from the excited to the ground state, by replacing ($\bar{n}_\mathrm{c} \to \bar{n}_\mathrm{c} + 1$). The extra term has a physical origin and corresponds to the vacuum field of the electromagnetic mode. It arises from the fact that the electromagnetic field is quantized , and has as any harmonic oscillator a zero point energy of $\hbar\omega/2$.

— Calculate with the new model the output power far below threshold of the laser, and neglect gain saturation for the low power regime. Use the normalized variables $x = \frac{\bar{N}_2}{\Delta N_t}$ and $y = \frac{N_T}{\Delta N_t}$, as well as $\bar{n}_\mathrm{sat}$for the photon saturation number, describing gain saturation.

— Derive the quadratic equation for the intracavity photon number in steady state, i.e. $\bar{n}_c$.

— Plot the solution of the intracavity photon number $\bar{n}_c$ versus pump for sample values, and show that functional form of the solution leads that at the lasing threshold, a dramatic increase in the intracavity photon number occurs. Plot the pump and cavity photon number on a log log plot.

4.3 Absorption of a single atom [1] Even a single atom can absorb all incident light of a laser beam. This counter-intuitive situation can be explained via interference of the atomic dipole emission and incoming light. This was experimentally demonstrated and

1. PRL 100, 093603 (2008) Single Photon Absorption by a Single Quantum Emitter

analyzed. Explain the conditions under which a single atom can absorb all the incident light. Explain the physical mechanism behind the atom absorbing all the incoming light. If no light is transmitted, where does the energy go ? Explain what happens if one measures the reflection of the atom. Next, what happens if there is also spontaneous emission present ?